JUNIOR GESTA MEETING 2016

The Junior GESTA will take place on April 27th-28th at UPC Barcelona (EPSEB, ground floor, Sala de Graus). This workshop is supported by a research grant provided by the Department of Mathematics UPC and by Generalitat de Catalunya 2014 SGR 00634 Geometria de varietats i aplicacions.

The organising committee: Álvaro del Pino, Anna Kiesenhofer, Eva Miranda, Arnau Planas, and Francisco Presas.

Accommodation

Rooms have been booked (between April 26th-28th) for most of the attendees at the following address:

Residencia Torre Girona.
Passeig dels Til·lers, 19.
08034 Barcelona.
Tlf: 93 390 4300

Breakfast is included.

The best way to reach the hotel from Barcelona's airport (El Prat), is by taking the metro. You will have to get off at Zona Universitaria stop (metro line 9).

Schedule

Wednesday, April 27

10:30-11:15	Roisin Braddell
11:15-11:45	*Coffee Break*
11:45-12:30	Anna Kiesenhofer
12:45-13:30	Abdó Roig
13:30-15:00	*Lunch*
15:00-15:45	Álvaro del Pino
15:45-16:15	*Coffee Break*
16:15-17:00	Cédric Oms
17:15-18:00	Samuel Ranz

Thursday, April 28

10:00-10:45	Damien Bouloc
10:45-11:30	Carles Sáez
11:30-12:00	*Coffee Break*
12:00-12:45	Rodrigo Schaefer
13:00-13:45	Jose Luis Perez
13:45-15:00	*Lunch*

On Thursday at 15:00 there is a class on dynamical systems by Amadeu Delshams at the EPSEB, room 0.4. Everyone is invited to attend.

Speakers and abstracts

Damien Bouloc: *The system of bending flows on the moduli space of 3D polygons*

The system of bending flows was introduced by Kapovich and Millson on the moduli space of 3D polygons with fixed side lengths. Geometrically, it corresponds to the bending of these polygons along fixed diagonals. For generic fixed side lengths, the moduli space is a symplectic manifold and we obtain a classical integrable Hamiltonian system. I will describe this system, and in particular its singular fibers. Indeed, even around degenerate singularities, the fibers are isotropic homogeneous submanifolds. Moreover we will not limit ourselves to generic side lengths, since most of the results can be actually adapted to the case when the moduli space is not a manifold.

Roisin Braddell: *The Virasoro-Bott Group and Equations of Shallow Water*

The Virasoro-Bott group is obtained as a central extension of the group of diffeomorphisms of the circle. Certain equations describing shallow water waves can be formulated as geodesic flows of right-invariant metrics of this group. These have similar properties and in particular are important examples of completely integrable systems. I will explore the Hamiltonian structure of these systems in detail.

Álvaro del Pino: *The space of convex curves in the 2-sphere*

It was proven by Hirsch and Smale that the space of immersions of a manifold into another one of greater dimension can be fully described by studying the space of "formal" immersions, whose computation is much easier. In Gromov's language, this means that immersions satisfy a full h-principle.

In a recent paper Saldanha solved the analogous question when one restricts to convex curves. This is readily seen to be a second order partial differential relation. Even though a full h-principle does not hold, he is able to describe the homotopy type of the space of convex curves in the 2-sphere in very geometric terms.

The aim of the talk will be to explain this beautiful result. If time allows, I will explain the role that convex curves play in some recent developments in Engel geometry.

Anna Kiesenhofer: *Action-angle coordinates on b-symplectic manifolds*

b-symplectic manifolds resemble symplectic manifolds, but degenerate along a hypersurface. In local coordinates a b-symplectic structure can be written $\frac{dx_1}{x_1} \wedge dy_1 + \sum_{i=2}^{n} dx_i \wedge dy_i$, where $2n$ is the dimension of the manifold. I will explain some basic geometric properties of these manifolds and discuss dynamics on them. More precisely, I will introduce b-integrable system and present an action-angle coordinate theorem, which is similar to the symplectic case.

The results are joint work with Eva Miranda and Geoffrey Scott.

José Luis Pérez: *Engel Geometry*

It has been shown recently, in joint work with R. Casals, Á. del Pino and F. Presas, that any flag in a 4-manifold can be homotoped to the flag induced by an Engel structure. We will start by reviewing some basic properties of Engel structures. Then, we shall state and prove a key lemma characterising the Engel condition locally, which will allow us to answer some classical questions.

Cédric Oms: *Existence of closed geodesics on closed manifolds*

I will describe the proof of existence of closed geodesics on closed manifolds. Morse theory will play a central role, as closed geodesics are critical points of the energy function. Care needs to be taken as the loop space fails to be compact. I will discuss a compactness substitute, known as the Palais–Smale condition. In the theorem, one needs to distinguish between two cases: an easier one when $\pi_1(M) \neq 0$, due to Cartan, and a harder one when $\pi_1(M) = 0$, known as the Lusternik–Fet theorem.

The proof is of historic importance, as it consists of one of the first successful links between variational methods and Riemannian geometry.

Samuel Ranz: *Geometric Invariant Theory and Symplectic Quotients*

In this talk we depict the link between geometric quotients arising in GIT and the Marsden-Weinstein quotient from symplectic geometry. To this effect, we begin with a reminder of algebraic varieties for those who were not familiar with this matter, followed with the construction of GIT quotients from algebraic geometry and symplectic quotients via moment maps. Finally, we prove the Kempf-Ness theorem, showing that, under some reasonable conditions, both quotients are homeomorphic.

Abdó Roig: *On the moduli space of algebraic cycles and their intersections*

In this talk I will describe the moduli space of algebraic cycles on a projective algebraic variety X, and how it can be used to produce interesting invariants of X. I will discuss some topics about the geometry and topology of these moduli spaces, with particular emphasis on results using the intersection product of cycles.

Carles Sáez: *Finite groups of symplectomorphisms*

I will talk about the following problem: Let (M, ω) be a symplectic manifold. Which are the finite subgroups of the group $\mathrm{Symp}(M, \omega)$? I will review what is known, including a complete answer to the problem in some cases (like $S^2 \times S^2$) and then I will explain some conjectures and ongoing work on this.

Rodrigo Schaefer: *Arnold diffusion using several combinations of Scattering maps*

In this work we illustrate the Arnold diffusion in a concrete example, the a priori unstable Hamiltonian system of $2 + 1/2$ degrees of freedom $H(p, q, I, \varphi, s) = \frac{p^2}{2} + \cos q - 1 + \frac{I^2}{2} + h(q, \varphi, s; \varepsilon)$. We prove that, for any small periodic perturbation of the form $h(q, \varphi, s; \varepsilon) = \varepsilon \cos q(a_{00} + a_{10} \cos \varphi + a_{01} \cos s)$, with $a_{10}a_{01} \neq 0$ and $\varepsilon \neq 0$ small enough, there is global instability for the action, i.e., $I(0) \leq -I(\varepsilon) < I(\varepsilon) \leq I(T)$ for some T and for any positive $I(\varepsilon) \leq C \log \frac{1}{\varepsilon}$ with C some constant. For this, we apply a geometrical mechanism based in the so-called Scattering map.

This work has the following structure: in a first stage, for the more restricted case $I(\varepsilon) \sim \pi/2\mu, \mu = \frac{a_{10}}{a_{01}}$, we use only one Scattering map. Later, in the general case we combine a Scattering map and the inner map (inner dynamics) to prove the main result (the existence of the instability for any μ). Finally, we consider multiple combinations of several scattering maps and we show different "ways of diffusion".